

Unit 2 Plan: Circular Trigonometry & Graphing

I. Unit Overview

- Unit Title: Unit 2: Radian Measure, the Unit Circle, & Graphing Trigonometric Functions
- Grade Level: 10th–12th Grade (Honors)
- Duration: 16 Instructional Days (Jan 20 – Feb 10)
- Key Concepts:
 - An angle is a unit of rotational measurement defined as the ratio of the length of an arc to the radius (radial measure).
 - A connection between geometric shapes and algebraic equations is created using a unit circle.
 - Trigonometric functions are mathematical expressions of circular movement characterized by parameters (such as amplitude, period, and phase shift).
- Essential Questions:
 - How does the unit circle generalize trigonometric functions beyond right triangles?
 - How does the translation of a function affect its periodic properties? . . .

II. Standards & Learning Targets

- MGSR 2.3: Convert between radians and degrees; solve problems using arc length and sector area formulas.
- MGSR 2.4-2.6: Label and interpret the 17-point unit circle; evaluate exact values of all six trig functions; explain symmetry and periodicity.
- PAFR 5.1: Graph trigonometric functions and their inverses; find key features (midline, amplitude, asymptotes, etc.).

III. Assessment Plan

- Diagnosis: Warm-up exercises on Radian-degree conversion and co-terminal angles (Day 1).
- Formative: Unit Circle Quiz (Day 7): A concentrated quiz testing your ability to label the 17-point unit circle and find exact values.
 - Derivita: Digital assessments with a target mastery level of 80%.
- Summative (Day 16): Unit 2 Test: All topics, including complex transformations and their applications.

IV. Instructional Sequencing

Day	Topic	Key Concepts & Hinge Questions	Resources
1	Radian Measure &	Converting θ ; Area of a Sector;	Packet 4.1

	Co-terminal Angles	Arc Length.	
2	Intro to the Unit Circle	Building the circle from special right triangles.	Packet 4.2
3-4	Evaluating Trig Functions	Using coordinates to find exact values for all 6 ratios.	Packet 4.2
5	Advanced Exact Values	Dealing with angles outside $[0, 2\pi]$ and co-terminal reasoning.	Packet 4.2
6	Quiz Review	Synthesizing unit circle and radian concepts.	Review pg. 30-39
7	Quiz: 4.1-4.2	<i>Mid-unit assessment (35 min)</i>	Quiz Materials
8-9	Inverse Trig Functions	Defining a domain form.	Inverse Videos
10	Graphing Sine Functions	$y = A \sin(B(x-C)) + D$; Period vs. Frequency.	Packet 4.5
11	Graphing Cosine Functions	Comparing sine and cosine phase shifts.	Packet 4.5
12-13	Reciprocal Graphs	Mapping vertical asymptotes for $\csc$ and $\sec$.	Packet 4.6
14	Tan & Cot Graphs	Identifying x -intercepts and vertical asymptotes.	Packet 4.6

15	Unit Review	Comprehensive analysis of all functions and transformations.	End of Packet
16	Unit 2 Test	<i>Summative Assessment</i>	Unit 2 Test

V. Differentiation & Scaffolding

- Scaffolding: Color-Coded Unit Circle: Apply blue for the cosine function (x) and red for the sine function (y).
 - Guided Templates for Graphing: Supply graphs that have “Five Critical Points” marked for sine and cosine functions.
- Extension: Reciprocal Graphing: Have students graph the sine function and find its x -intercepts; then plot the vertical asymptotes of the cosecant function accordingly.
 - Analysis of Function Transformation: Analyze why tangent’s period is π and sine’s period is 2π .

VI. Instructional Strategies

- Hinge Point Check: After the first half of the 90-minute lesson, conduct a "Quick-Fire" exercise for exact value problems (for example, "What is $\sin \frac{5\pi}{6}$ "), gauging preparedness prior to continuing to graphing.
- Dual Coding: Link the algebraic expression $y = A \sin(Bx)$ with a visual representation on the Promethean board where vertical stretches of the graph are compared to an increased value of A.

LESSON # 2	
Subject/Topic: Math/pre-calculus	Grade Level: 10 th , 11 th , 12 th
Lesson Structure or Grouping: Whole Class ☒ Small Group ☐ 1:1 ☐ Other (specify):	Learning Segment Theme: Right Triangle Trig: Exact vs. Approximate
Resources and Materials: Note packet, videos, extra practice, calculator, iPad, Promethean board, pencil, colored markers	
Standards: State: PC.FT.2: Define trigonometric ratios (sin, cos, tan, csc, sec, cot) for acute angles in right triangles. PC.FIF.7: Graph functions showing period, midline, and stretch, and use precision in mathematical modeling. National: HSF.TF.A.3: Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$, and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent in terms of their coordinates. HSF.IF.C.7.e: Graph trigonometric functions, showing period, midline, and amplitude.	
Objective: - Students will accurately calculate trigonometric ratios for acute angles in right triangles and express results in both simplified radical form (exact) and rounded decimal form (approximate). Students will demonstrate - conceptual understanding of triangle structure and precision, - procedural fluency in selecting SOHCAHTOA ratios, simplifying radicals, and rounding correctly, and - mathematical reasoning by justifying when an exact versus approximate value is appropriate. Mastery is met when students correctly solve at least 2 of 3 problems using both forms, use the $\approx$ symbol appropriately, and provide a brief justification for their chosen representation.	
ACADEMIC LANGUAGE DEMANDS	
Language Demands: Students must distinguish between the instruction to find an "exact" value versus an "approximate" value in word problems and mathematical modeling	
Language Supports: - SOHCAHTOA mnemonic to help students consistently choose the correct ratio. - Color-coded notes showing exact vs. approximate values. - Sentence frames prompt students to explain which ratio they chose and why. - Eye-level triangle diagrams labeled Opposite/Adjacent/Hypotenuse	
Essential Vocabulary: Exact value, approximation, radical, irrational number, rounding, ratio.	
INSTRUCTION	
Direct Instruction/Modeling: - Establish the conceptual difference between exact (radical) and approximate (decimal) values and explain why rounding introduces precision loss. - Revisit the Pythagorean Theorem for ground-side relationships. - Model how to identify a <i>complete ratio</i> by labeling sides in relation to a given angle,	

then selecting the correct trig function.

4. Using a right triangle with sides $a = 3$, $b = 4$, show $c = 5$ and compute all six trig functions in both exact and approximate forms, using color-coding to highlight the difference.
5. Explicitly connect these ratios to upcoming **unit circle** coordinates to support long-term conceptual understanding.

Guided Practice:

1. Work through a problem as a class where the hypotenuse is not a whole number. For example, if legs are $a=1$ and $b=2$, lead students to calculate $c = \sqrt{(1^2 + 2^2)} = \sqrt{5}$.
2. Ask students to identify the **Exact** answer ($\sqrt{5}$) and the **Approximate** answer using their calculators (e.g., 2.24 to the nearest hundredth).
3. Use "Example 1" from the Unit 3 Packet where side measurements are provided for a marked angle θ
4. Guide students in labeling the sides as "Opposite," "Adjacent," or "Hypotenuse" relative to θ before calculating ratios.
5. Direct students to solve a missing side in a real-world context, such as a surveyor's measurement, and practice rounding to the nearest tenth or nearest whole number as specified in the prompt.

Independent Practice:

1. *You Try 1* ($a = 5$, $b = 12$): Find the hypotenuse and all six trig functions using both exact and approximate values.
2. *You Try 2*: Reinforce ratio selection when the triangle's orientation changes.
3. Complete angle of elevation/depression tasks (e.g., building height from 300 ft at 22° ; ladder safety using a 110-ft ladder at 72°).
4. For **every** problem, students must:
 - a. Give exact and approximate values,
 - b. Use the $\approx$ symbol correctly, and
 - c. Write a sentence justifying their choice of representation.

Closure:

1. Ask students why engineers (e.g., in bridge design) keep values exact until the final step.
2. Provide a triangle with legs 3 and 6; students calculate the hypotenuse in exact and approximate form and explain the impact of rounding.
3. Preview: These exact ratios will appear as coordinates on the **unit circle** in the next lesson.

CONNECTION TO RESEARCH

Research Base (Theory):

Dual Coding Theory (Paivio, 2006). This theory posits that the human mind processes information through two separate but linked channels: one for verbal text and one for visual imagery. By presenting mathematical concepts through both symbolic notation and visual structures, students create dual "memory traces" that significantly improve long-term retention and retrieval.

Research Application to Lesson:

A T-chart is used to visually anchor the distinction between "Exact" and "Approximate" values, forcing students to spatially categorize radicals as precise symbols and decimals as estimations. This visual organization reduces cognitive load by providing a clear "mental map" for students to follow when deciding which mathematical form is required for a specific problem.

APA research citation:

Paivio, A. (2006). *Dual Coding Theory and Education*. Draft Chapter for the Conference on Educational Psychology.

DIFFERENTIATION

Supporting Students with Special Needs: Provide a "Square Root Cheat Sheet" containing common perfect squares and their simplified radicals to reduce the cognitive load during the calculation of exact values.

Challenging Above-Average Students:

Provide a *Square Root Cheat Sheet*, partially completed diagrams, and color cues to reduce cognitive load and support conceptual understanding.

Facilitating a Classroom Environment that Supports Student Learning: Establish "Calculator-Free" zones during the first 10 minutes of the lesson to ensure students focus on the conceptual derivation of exact ratios before relying on decimal approximations.

Extension: Have students explore complementary angles to show that $\sin(A) = \cos(B)$; maintaining exact values reveals structure obscured by decimals.

ASSESSMENT

Diagnostic/Pre-Assessment: "Warm-up" question: Find the decimal for $1/3$ and discuss why 0.33 is not "exact."

Formative Assessment: Check for correct use of $\approx$ vs. $=$, correct ratio selection and labeling, and quality of written justification for exact/approximate choice.

Summative Assessment: Derivita with 80% accuracy